GCE Examinations

Decision Mathematics
Module D2

Advanced Subsidiary / Advanced Level

Paper C

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Craig Hunter, Edited by Shaun Armstrong

1. *This question should be answered on the sheet provided.*

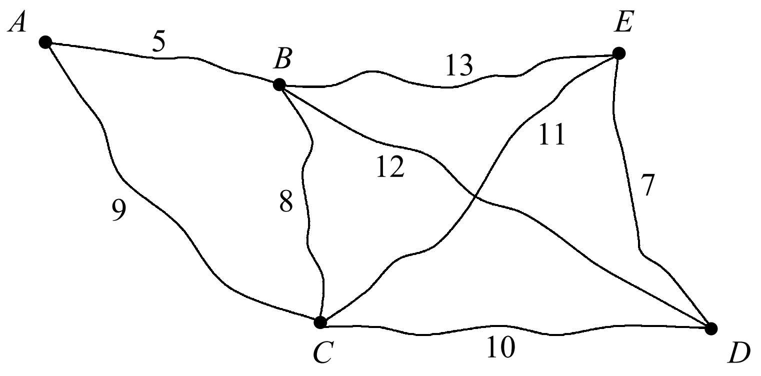

Fig. 1

The network in Figure 1 shows the distances, in miles, between the five villages in which Sarah is planning to enquire about holiday work, with village A being Sarah's home village.

(a) Illustrate this situation as a complete network showing the shortest distances. **(2 marks)**

(b) Use the nearest neighbour algorithm, starting with A, to find an upper bound to the length of a tour beginning and ending at A. **(2 marks)**

(c) Interpret the tour found in part *(b)* in terms of the original network. **(2 marks)**

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2. The payoff matrix for player A in a two-person zero-sum game with value V is shown below.

		B		
		I	II	III
A	I	$^-1$	4	$^-3$
	II	$^-3$	7	1
	III	5	$^-2$	$^-1$

Formulate this information as a linear programming problem, the solution to which will give the optimal strategy for player B.

(a) Rewrite the matrix as necessary and state the new value of the game, v, in terms of V. **(2 marks)**

(b) Define your decision variables. **(2 marks)**

(c) Write down the objective function in terms of your decision variables. **(2 marks)**

(d) Write down the constraints. **(2 marks)**

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3. *This question should be answered on the sheet provided.*

Arthur is planning a bus journey from town A to town L. There are various routes he can take but he will have to change buses three times – at B, C or D, at E, F, G or H and at I, J or K.

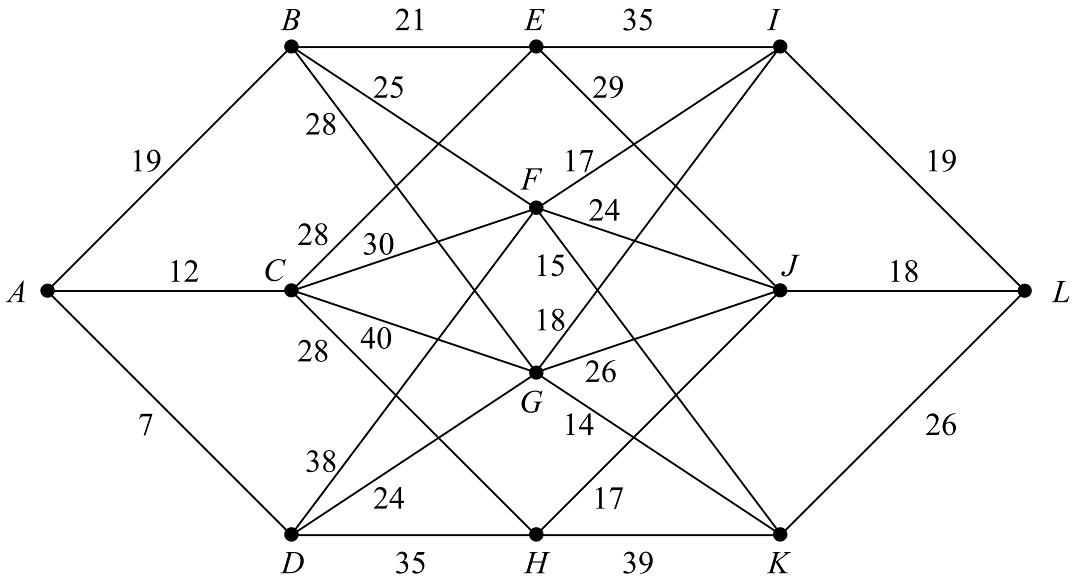

Fig. 2

Figure 2 shows the bus routes that Arthur can use. The number on each arc shows the average waiting time, in minutes, for a bus to come on that route. As the forecast is for rain, Arthur wishes to plan his journey so that the maximum waiting time at any one stop is as small as possible.

Use dynamic programming to find the route that Arthur should use. **(9 marks)**

Turn over

4. A furniture manufacturer has three workshops, W_1, W_2 and W_3. Orders for rolls of fabric are to be placed with three suppliers, S_1, S_2 and S_3. The supply, demand and cost per roll in pounds, according to which supplier each workshop uses, are given in the table below.

	W_1	W_2	W_3	Available
S_1	12	11	17	30
S_2	7	5	10	25
S_3	5	6	8	10
Required	20	15	30	

Starting with the north-west corner method of finding an initial solution, find an optimal transportation pattern which minimises the total cost. State the final solution and its total cost.

(11 marks)

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5. A travel company offers a touring holiday which stops at four locations, A, B, C and D. The tour may be taken with the locations appearing in any order, but the number of days spent in each location is dependent on its position in the tour, as shown in the table below.

	Stage			
	1	**2**	**3**	**4**
A	7	8	5	6
B	6	9	6	5
C	9	8	5	7
D	7	7	6	6

Showing the state of the table at each stage, use the Hungarian algorithm to find the order in which to complete the tour so as to maximise the total number of days. State the maximum total number of days that can be spent in the four locations.

(11 marks)

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6. The payoff matrix for player A in a two-person zero-sum game is shown below.

		B		
		I	II	III
A	I	3	5	$^-2$
	II	7	$^-4$	$^-1$
	III	9	$^-4$	8

(a) Applying the dominance rule, explain, with justification, which strategy can be ignored by

(i) player A,

(ii) player B. **(3 marks)**

(b) For the reduced table, find the optimal strategy for

(i) player A,

(ii) player B. **(8 marks)**

(c) Find the value of the game. **(2 marks)**

Turn over

7. *This question should be answered on the sheet provided.*

A tinned food producer delivers goods to six supermarket warehouses, *B*, *C*, *D*, *E*, *F* and *G*, from its base, *A*. The distances, in kilometres, between each location are given in the table below.

	A	*B*	*C*	*D*	*E*	*F*	*G*
A	–	63	75	57	81	102	52
B	63	–	48	83	32	49	61
C	75	48	–	41	72	65	109
D	57	83	41	–	49	79	70
E	81	32	72	49	–	51	88
F	102	49	65	79	51	–	90
G	52	61	109	70	88	90	–

The producer wishes to plan a route beginning and ending at *A* which visits each of the warehouses.

(a) Obtain and draw a minimum spanning tree for this network and hence calculate an upper bound for the total length of the delivery journey.

(6 marks)

(b) Using two shortcuts, improve this upper bound to find an upper bound of less than 375 kilometres.

(3 marks)

(c) By deleting *G*, find a lower bound for the total length of the journey. **(3 marks)**

(d) By deleting *B*, obtain another lower bound for the total length of the journey.

(4 marks)

(e) State, with a reason, which of the lower bounds found in parts *(c)* and *(d)* is the better lower bound.

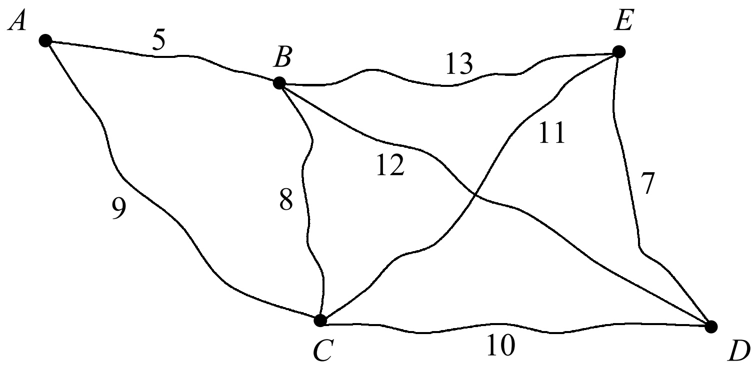
(1 mark)

END

Sheet for answering question 1 *NAME* ....

Please hand this sheet in for marking

(a)



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(b)

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(c) ....

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Stage	State	Action		
1	*I*	*L*		
	J	*L*		
	K	*L*		
2	*E*	*I* *J*		
	F	*I* *J* *K*		
	G	*I* *J* *K*		
	H	*J* *K*		
3	*B*	*E* *F* *G*		
	C	*E* *F* *G* *H*		
	D	*F* *G* *H*		
4	*A*	*B* *C* *D*		

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Sheet for answering question 7 NAME ……………………………………….

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(a)

	A	*B*	*C*	*D*	*E*	*F*	*G*
A	–	63	75	57	81	102	52
B	63	–	48	83	32	49	61
C	75	48	–	41	72	65	109
D	57	83	41	–	49	79	70
E	81	32	72	49	–	51	88
F	102	49	65	79	51	–	90
G	52	61	109	70	88	90	–

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(b) ………………………………………………………………………………………………

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Turn over

Sheet for answering question 7 (cont.)

(c)

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(d)

	A	*B*	*C*	*D*	*E*	*F*	*G*
A	–	63	75	57	81	102	52
B	63	–	48	83	32	49	61
C	75	48	–	41	72	65	109
D	57	83	41	–	49	79	70
E	81	32	72	49	–	51	88
F	102	49	65	79	51	–	90
G	52	61	109	70	88	90	–

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(e) ……………………………………………………………………………………………

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